

## 104 學年度四技二專第一次聯合模擬考試 共同科目 數學(A)卷 詳解

數學(A)卷

104-1-A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	C	C	C	B	A	D	A	D	D	D	A	C	C	A	B	D	B	B	A	B	B	A	C	C

1.  $\because M$  為  $\overline{AB}$  的中點

$$\therefore \frac{x+(-1)}{2} = 1, \frac{-3+y}{2} = -2$$

$$\Rightarrow x = 3, y = -1$$

$$\Rightarrow x + y = 2, xy = -3$$

故點  $P(x+y, xy)$  在第四象限

2.  $\overline{AB} = \sqrt{(2-3)^2 + (-2-5)^2} = \sqrt{50} = 5\sqrt{2}$

$$\overline{AP} = \frac{3}{5}\overline{AB} = \frac{3}{5} \times 5\sqrt{2} = 3\sqrt{2}$$

3.  $m = \frac{k-7}{6-3} = -3 \Rightarrow k-7 = -9 \Rightarrow k = -2$

4. 原式 =  $(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{\sqrt{3}}{2})^2 + 1^2 = \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = \frac{5}{2}$

5.  $2015^\circ = 5 \times 360^\circ + 215^\circ$  為第三象限角

$$\therefore \sin 2015^\circ < 0, \tan 2015^\circ > 0$$

故點  $P(\sin 2015^\circ, \tan 2015^\circ)$  在第二象限

6.  $x$  截距為 2, 即過點  $(2, 0)$

由點斜式知直線方程式為

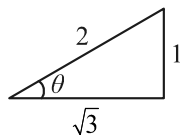
$$y - 0 = 3(x - 2) \Rightarrow y = 3x - 6$$

故由斜截式知  $y$  截距為 -6

7. 原式  $\Rightarrow 1 + \sin \theta = 3 - 3 \sin \theta$

$$\Rightarrow \sin \theta = \frac{1}{2}, \text{作圖如右:}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}}$$



8. 過點  $A(0, -3)$  即  $y$  截距為 -3, 設  $x$  截距為  $a$

由截距式知直線方程式為  $\frac{x}{a} + \frac{y}{-3} = 1$

$$\because \text{過點}(2, -6) \text{ 代入得 } \frac{2}{a} + \frac{-6}{-3} = 1 \Rightarrow a = -2$$

$\therefore$  直線與兩坐標軸所圍成的三角形面積為

$$\frac{1}{2} \times |(-2) \times (-3)| = 3$$

9.  $\overline{AB} = 3\overline{BC} \Rightarrow \overline{AB} : \overline{BC} = 3 : 1$

$$A(2, -5) \quad B(-1, y) \quad C(x, 3)$$

由分點坐標得  $-1 = \frac{3x+2}{3+1}, y = \frac{9-5}{3+1}$

$$\Rightarrow x = -2, y = 1, \therefore x + y = -1$$

10.  $a : b : c = \sin A : \sin B : \sin C = 2 : 3 : 4$

$$\cos C = \frac{2^2 + 3^2 - 4^2}{2 \times 2 \times 3} = -\frac{1}{4}, \sec C = -4$$

11. 兩邊平方得  $(\cos \theta - \sin \theta)^2 = \frac{1}{4}$

$$\Rightarrow 1 - 2 \sin \theta \cos \theta = \frac{1}{4} \Rightarrow \sin \theta \cos \theta = \frac{3}{8}$$

12.  $\tan(3\pi - \frac{\pi}{3}) = \tan(\pi - \frac{\pi}{3}) = -\tan \frac{\pi}{3} = -\sqrt{3}$

13. 原式 =  $\frac{2}{1 + \sin 20^\circ} + \frac{2}{1 + \frac{1}{\sin 20^\circ}}$

$$= \frac{2}{1 + \sin 20^\circ} + \frac{2}{\frac{\sin 20^\circ + 1}{\sin 20^\circ}} = \frac{2}{1 + \sin 20^\circ} + \frac{2 \sin 20^\circ}{\sin 20^\circ + 1}$$

$$= \frac{2(1 + \sin 20^\circ)}{1 + \sin 20^\circ} = 2$$

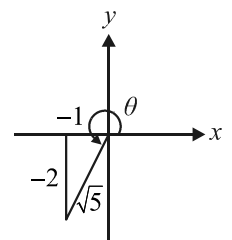
14.  $\because \tan \theta = 2$  且  $\sin \theta < 0$

$\therefore \theta$  為第三象限角, 作圖如右:

故  $3 \sin \theta - \cos \theta$

$$= 3 \times (\frac{-2}{\sqrt{5}}) - (-\frac{1}{\sqrt{5}})$$

$$= \frac{-5}{\sqrt{5}} = -\sqrt{5}$$



15.  $\therefore \begin{cases} f(2) = 2a + b = -11 \\ f(4) = 4a + b = -21 \end{cases}$

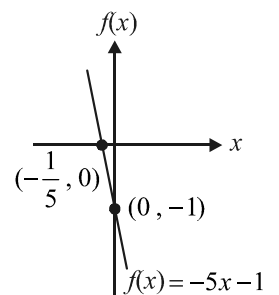
$$\Rightarrow a = -5, b = -1$$

$$\therefore f(x) = -5x - 1$$

$x$	0	$-\frac{1}{5}$
$f(x)$	-1	0

作圖如右:

故圖形不通過第一象限



16. 設扇形半徑為  $r$ , 圓心角為  $\theta$

$$\begin{cases} S = r\theta = 5 \\ A = \frac{1}{2}r^2\theta = 10 \end{cases} \Rightarrow r = 4, \theta = \frac{5}{4}$$

17.  $L_1 : x = -3y + 1 \Rightarrow m_1 = -\frac{1}{3}$

$$L_2 : y = mx + 3 \Rightarrow m_2 = m$$

$$\because L_1 \perp L_2 \Rightarrow m_1 \times m_2 = -1 \Rightarrow (-\frac{1}{3}) \times m = -1 \Rightarrow m = 3$$

18.  $\Delta ABC$  面積

$$= \frac{1}{2} \times 2 \times 4\sqrt{3} \times \sin 60^\circ = \frac{1}{2} \times 2 \times 4\sqrt{3} \times \frac{\sqrt{3}}{2} = 6$$

19.  $\therefore \cos x$  的週期為  $2\pi$

$$\therefore f(x) \text{ 的週期為 } \frac{2\pi}{3}$$

20.  $a = \sin 100^\circ = \sin(180^\circ - 80^\circ) = \sin 80^\circ$

$$b = \sin 200^\circ = \sin(180^\circ + 20^\circ) = -\sin 20^\circ$$

$$c = \sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ$$

$$\therefore a > b > c$$

21.  $\therefore -1 \leq \cos x \leq 1$

$\therefore$  當  $\cos x = -1$  時， $f(x)$  有最小值

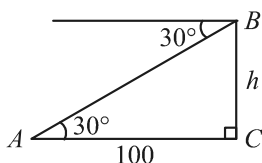
$$2 - \left(-1 - \frac{1}{2}\right)^2 = -\frac{1}{4}$$

22. 設樓高為  $h$  公尺

$$\text{則 } \tan 30^\circ = \frac{h}{100} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = 100$$

$$\Rightarrow h = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$



23. 設所求直線為  $x - 2y + t = 0$

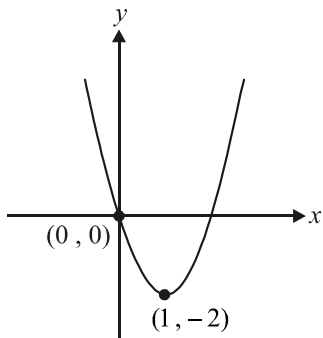
$$\therefore \text{過點}(1, 2) \text{ 代入得 } 1 - 4 + t = 0 \Rightarrow t = 3$$

$$\therefore \text{所求直線為 } x - 2y + 3 = 0 \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$$

$$\text{故 } m = \frac{1}{2}, k = \frac{3}{2}, m - k = \frac{1}{2} - \frac{3}{2} = -1$$

24.  $\therefore$  圖形頂點為  $(1, -2)$ ，且開口向上

又  $f(0) = 0$ ，即與  $y$  軸交於原點，作圖如下：



故圖形不通過第三象限

25.  $\overline{AC}^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 60^\circ = 7$ ， $\overline{AC} = \sqrt{7}$

設  $\triangle ABC$  之外接圓半徑為  $R$

$$\frac{\overline{AC}}{\sin B} = 2R \Rightarrow \frac{\sqrt{7}}{\sin 60^\circ} = 2R$$

$$\Rightarrow \frac{\sqrt{7}}{\frac{\sqrt{3}}{2}} = 2R \Rightarrow R = \frac{\sqrt{7}}{\sqrt{3}}$$

$$\text{外接圓面積為 } \pi R^2 = \frac{7}{3}\pi$$